Teaching the Arithmetic of Infinity

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- An Amazonian tribe called Pirahã has numerical symbols to denote only 1 and 2.
- Any greater quantity is referred to as ‘many’. Suppose that, instead of ‘many’, we introduced the symbol ∞.
- Then, in Pirahã arithmetic, we would have $1 + 1 = 2$ and $1 + 2 = \infty = 2 + 2$.

- Sometimes we can carry out computations with ∞ but at other times we can't, because of indeterminate forms.
- The Pirahã would denote the size of both a collection of five objects and a subcollection of four by the symbol ∞.
- Then $\infty - \infty = 1$, but only in this case! For instance, with a subcollection of three objects $\infty - \infty = 2$.
- With a collection of nine objects and a subcollection of five respectively, we get $\infty - \infty = \infty$.

- Indeterminate forms arise because we are not able to make enough distinctions of size.
- Y. Sergeyev (2003, 2008, 2009) has pointed out that we are in the same situation as the Pirahã with respect to infinity.
- Because we cannot discriminate between sizes of infinity when we use ∞, we cannot calculate properly with this symbol.
- This is also true of set theory: cardinal numbers do not lead to enough discriminations to eliminate indeterminate forms.

- Consider $\mathbb{N} = \{1, 2, 3, \ldots\}$. Let us introduce a numerical symbol that indicates its size.
- This is ① (called 'grossone'). We assume ① $> n$ for finite n and that we can do arithmetic with ①.
- E.g. we have: ① $- 1 <$ ① $<$ ① $+ 2 <$ ①^2.
- When we use ∞, we cannot draw any distinctions between the terms in the above inequalities.

- Note that ① expresses the size of $\mathbb{N}$. The size of $\mathbb{N} - \{1\} = \{2, 3, \ldots\}$ is expressed by $① - 1$.
- The size of $\mathbb{N} \cup \{\sqrt{2}\}$ is expressed by $① + 1$.
- The set of all pairs of natural numbers, of the form $\langle m, n \rangle$ has an infinite size expressed by $①^2$.
- The size of the collection of positive integers, negative integers and 0 is $2① + 1$.
- In set theory, all of these collections are indistinguishable relative to size.

- These are simple ideas, with far reaching consequences (we shall explore more later, in the workshop).
- They have been applied to the calculus, ordinary differential equations, fractals, set theory.
- They have been applied to cellular automata, operations research, material science, computability theory.
- The Infinity Computer, which can work with ①, has been patented and will be mass-produced.


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- These ideas offer an accessible and rewarding enrichment opportunity for all students.
- They can be used to develop geometric series, differentiation and integration in a very simple and powerful way.
- They have clear ties with further maths, for example with linear systems and decision problems.
- We hope that you can tell us, by the end of today, where else they might be helpful and in what ways.

References

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